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# Piecewise Linear Two-Dimensional Warping

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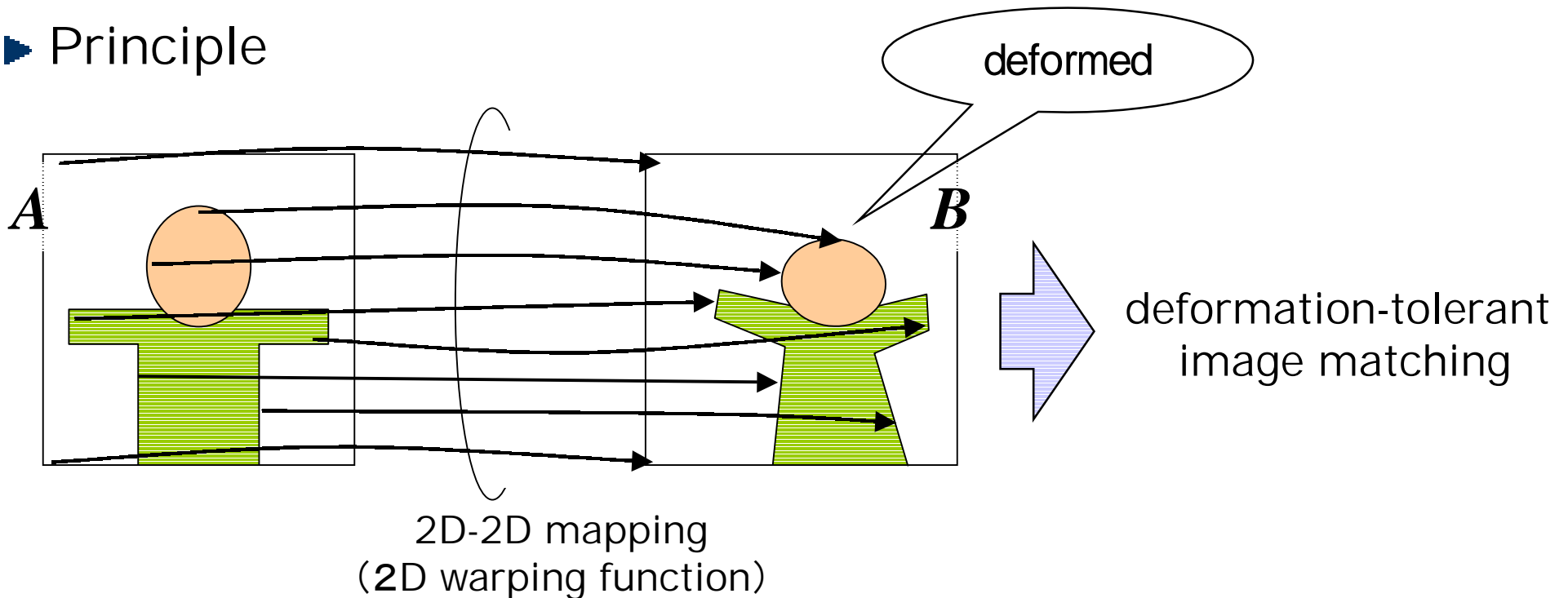
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# Elastic Image Matching

## ▶ Principle

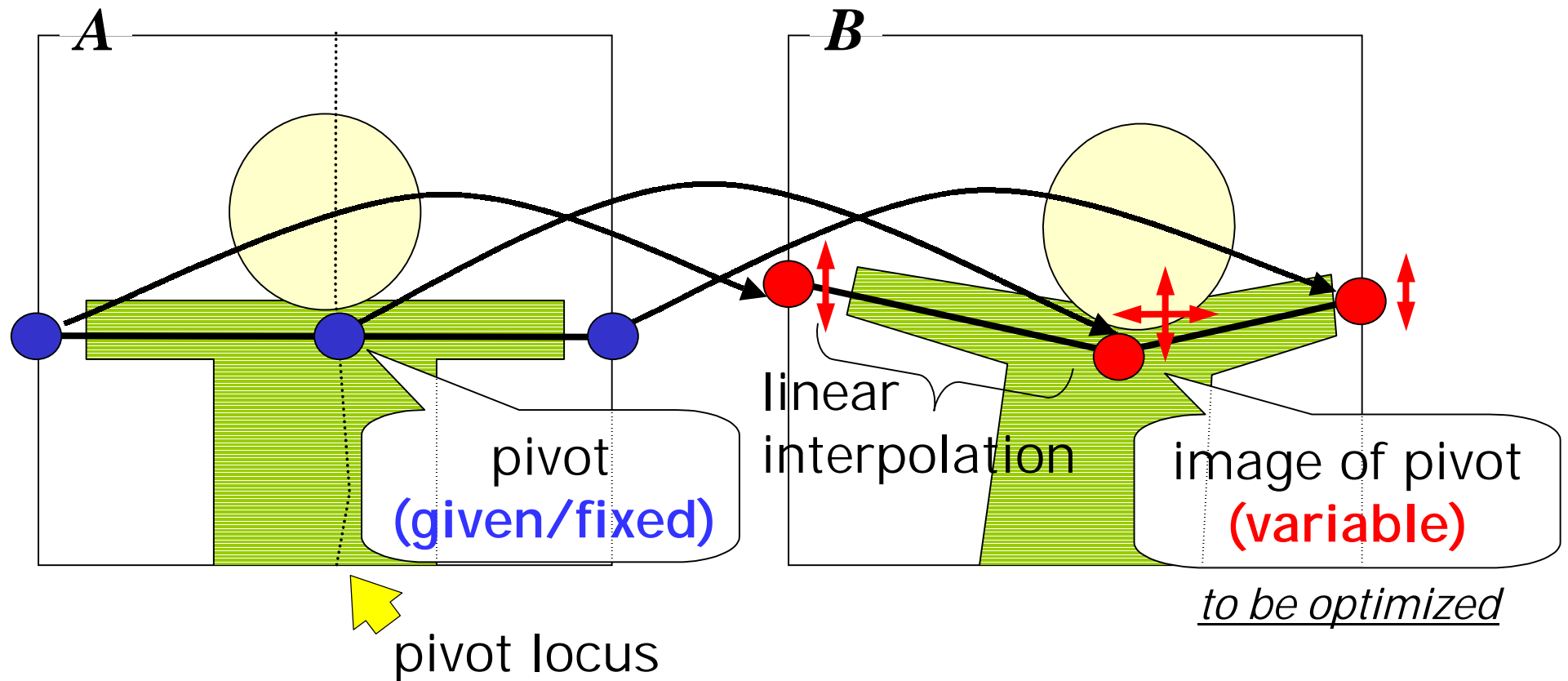


## ▶ Two central problems

- Formulation of warping function
- Algorithm to obtain the optimal warping function which gives the best match between given images

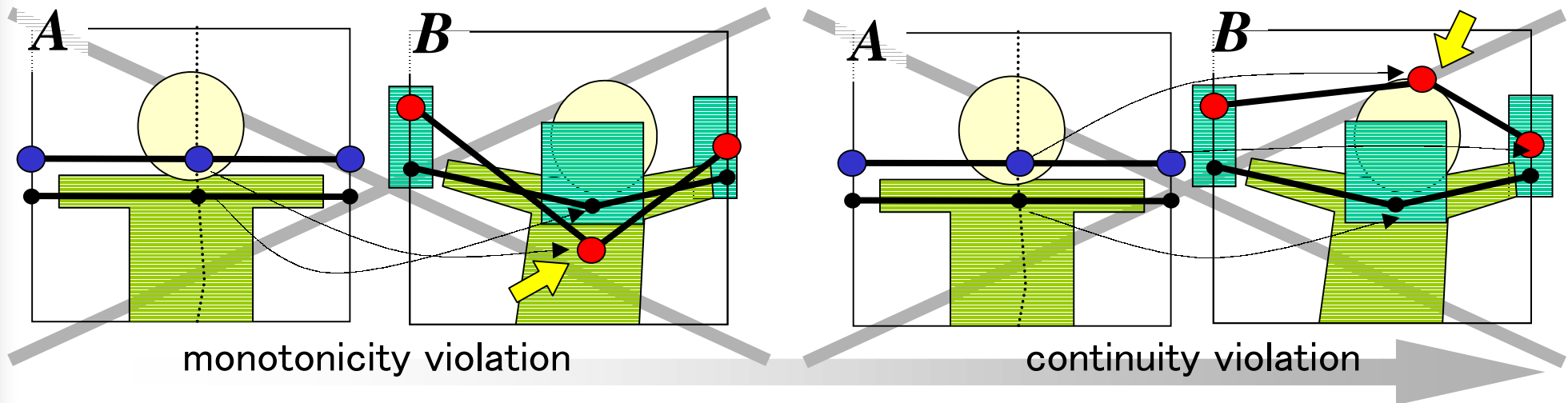
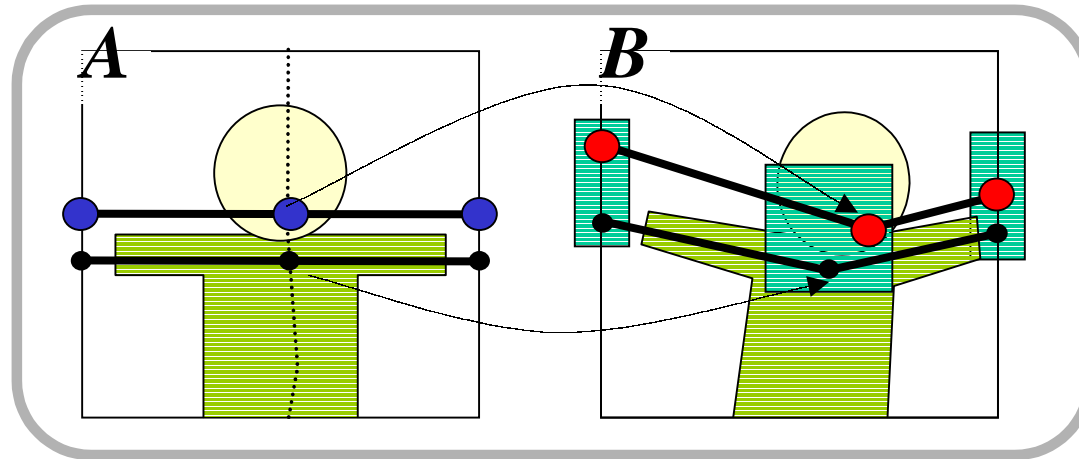
# Formulation of Piecewise Linear 2DW (1)

- ▶ Each column of  $A$  is mapped to  $B$  as a broken line



# Formulation of Piecewise Linear 2DW (2)

- ▶ Monotonicity and continuity constraints for topology preservation



# Examples (1)



image *A*

pivot loci  
on *A*

image *B*

warp

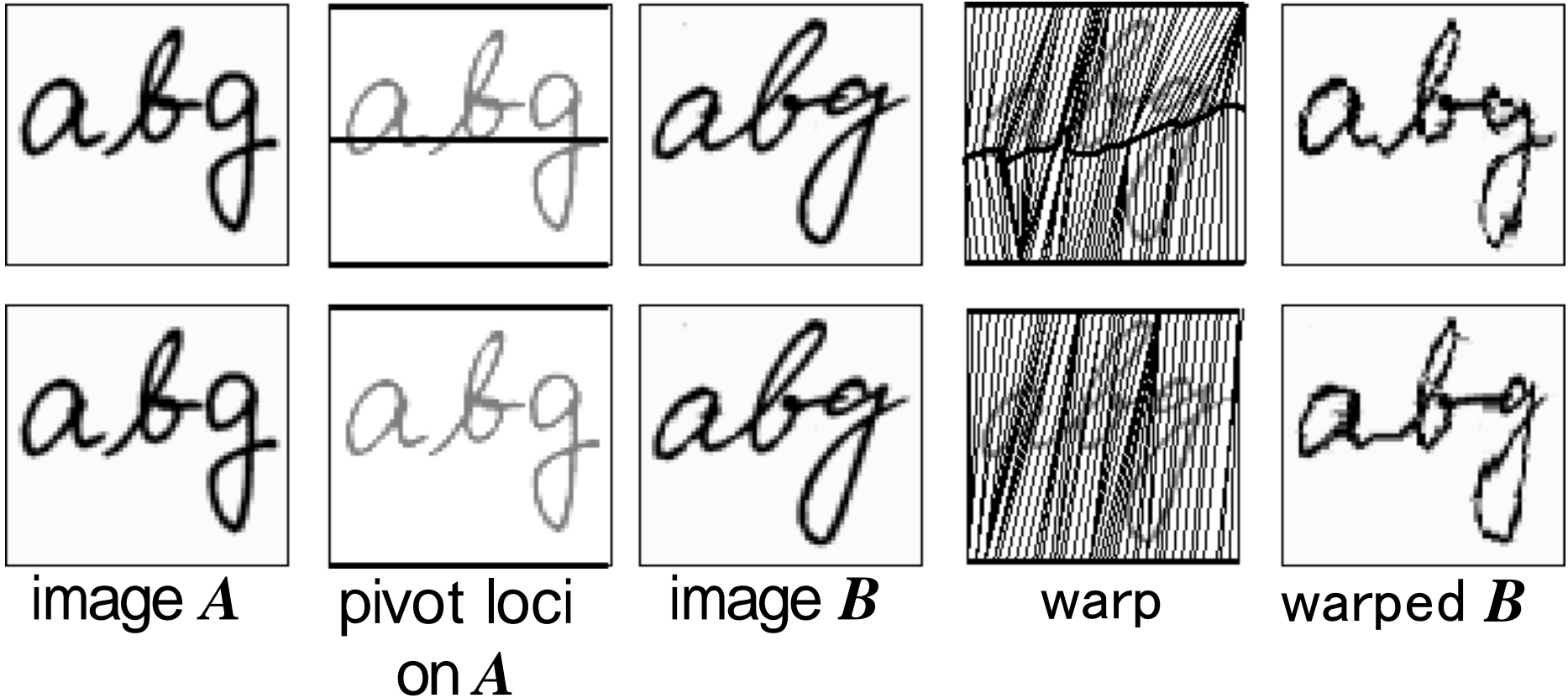
warped *B*

given images

results



# Examples (2)



given images

results



# Examples (3)

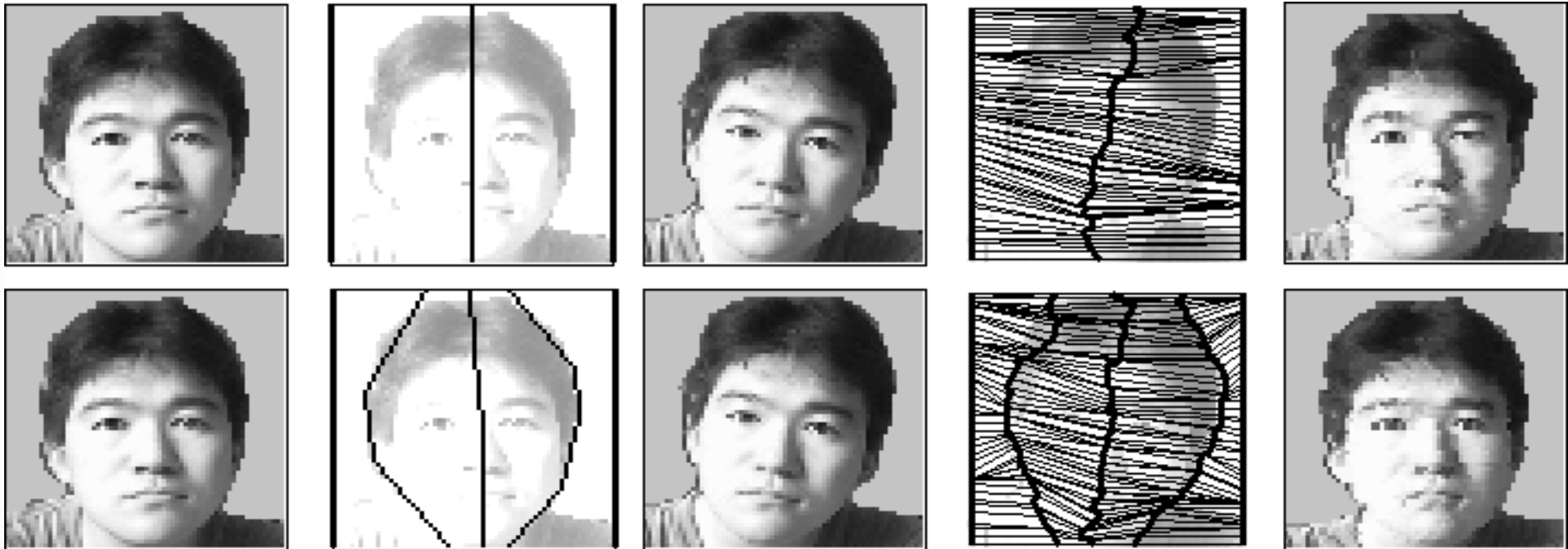


image *A*

pivot loci  
on *A*

image *B*

warp

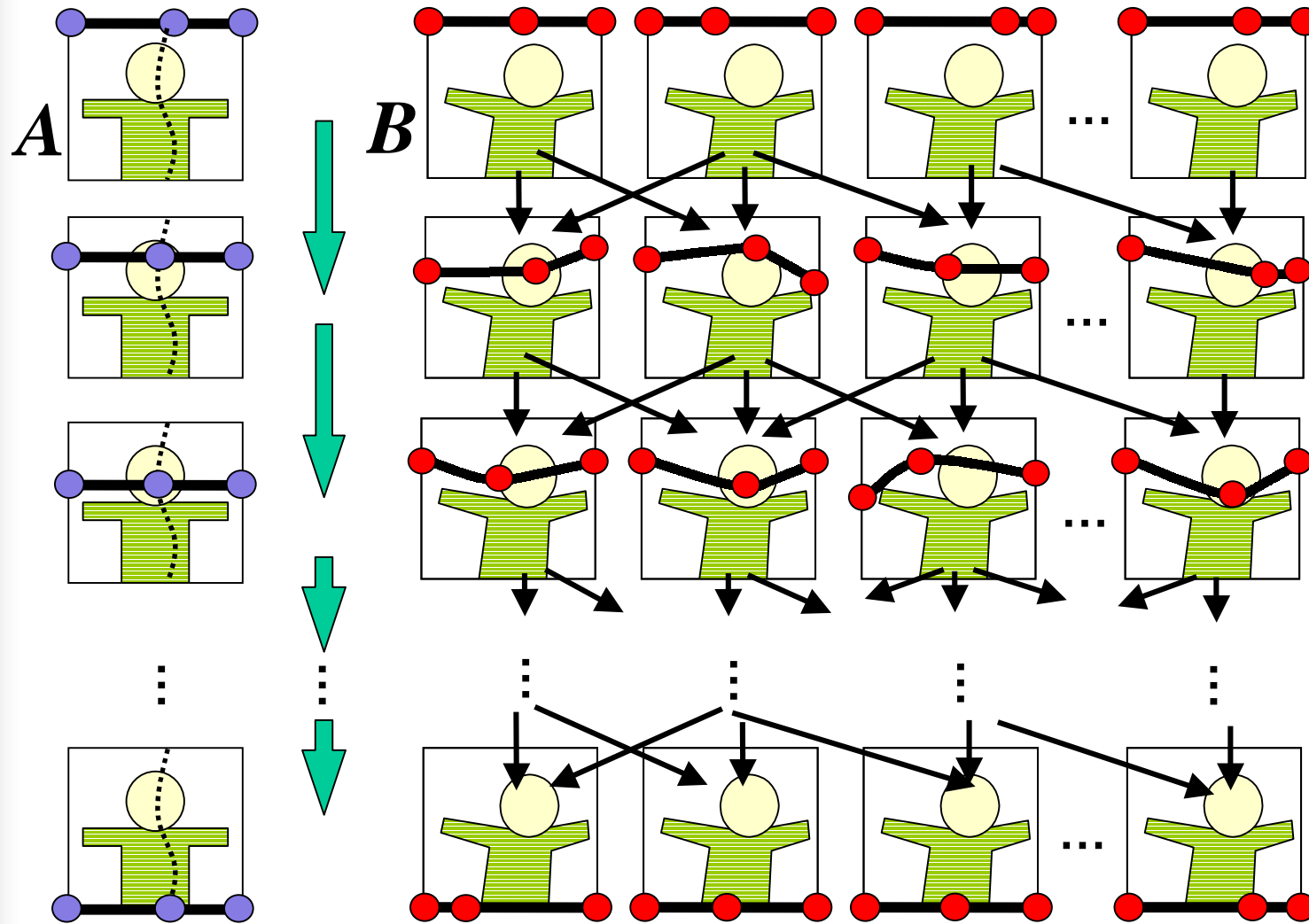
warped *B*

given images

results



# The DP Algorithm : A Graphical Representation



2DW optimization problem

Row-wise organization

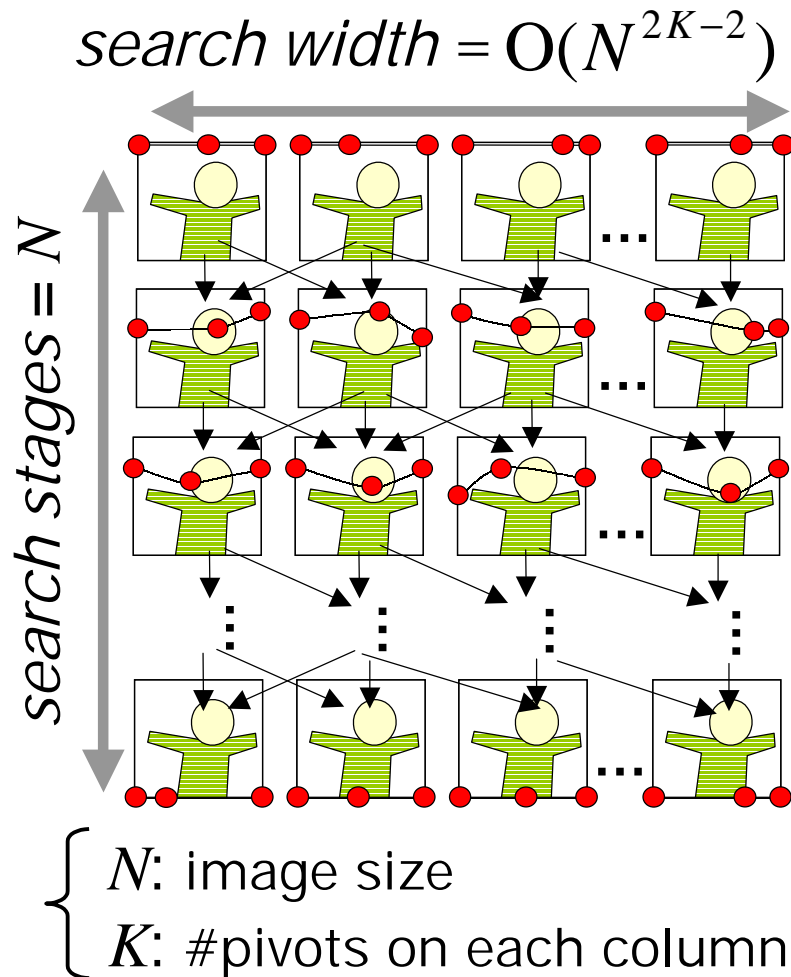
Best path problem

Efficiently solvable by DP





# Computational Complexity



time complexity

$$\approx O(N^{2K-1}(9^K + N))$$

space complexity

$$\approx O(N^{2K-1})$$

# Complexity Reduction to Use More Pivots

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## ▶ Re-organizing the DP Algorithm

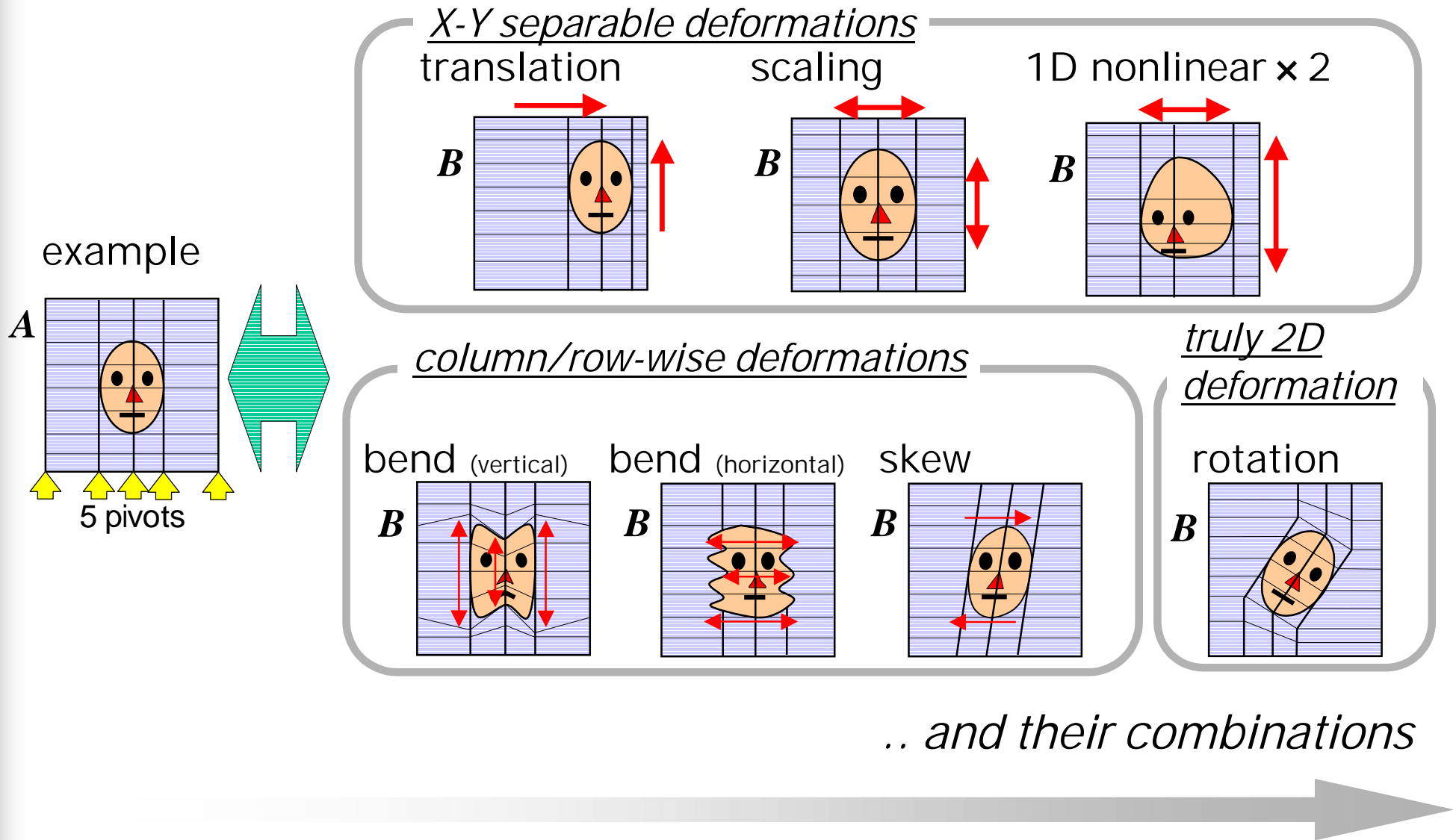
- Use *Pixel-wise* organization instead of the row-wise organization
- Still remains in exponential order of #pivots,  $K$

## ▶ Applying Beam Search

- *Pruning off* less hopeful search paths
- Suboptimal solution is obtained in polynomial time

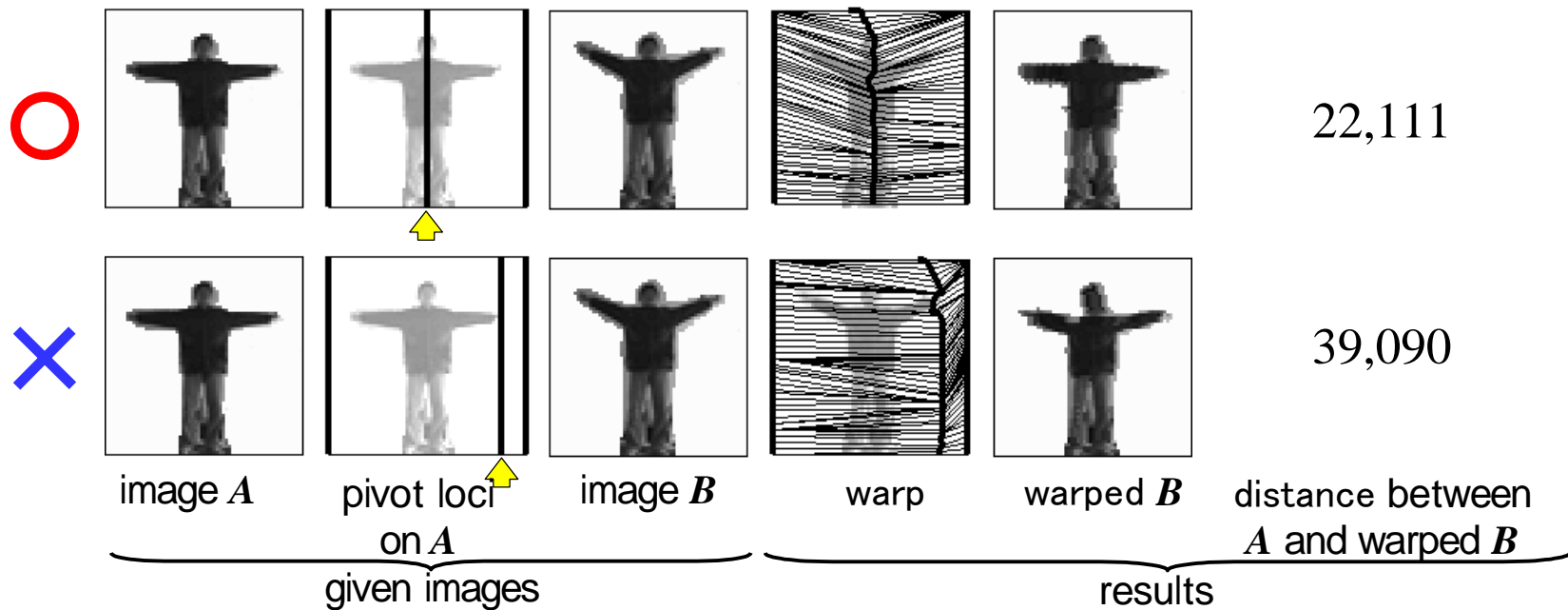


# Discussion (1) : Admissible Deformation



# Discussion (2) : Where to Place the Pivots?

- ▶ Pivots should be placed on the points considering deformation characteristic of target



- ▶ Distortion due to linear interpolation should also be considered



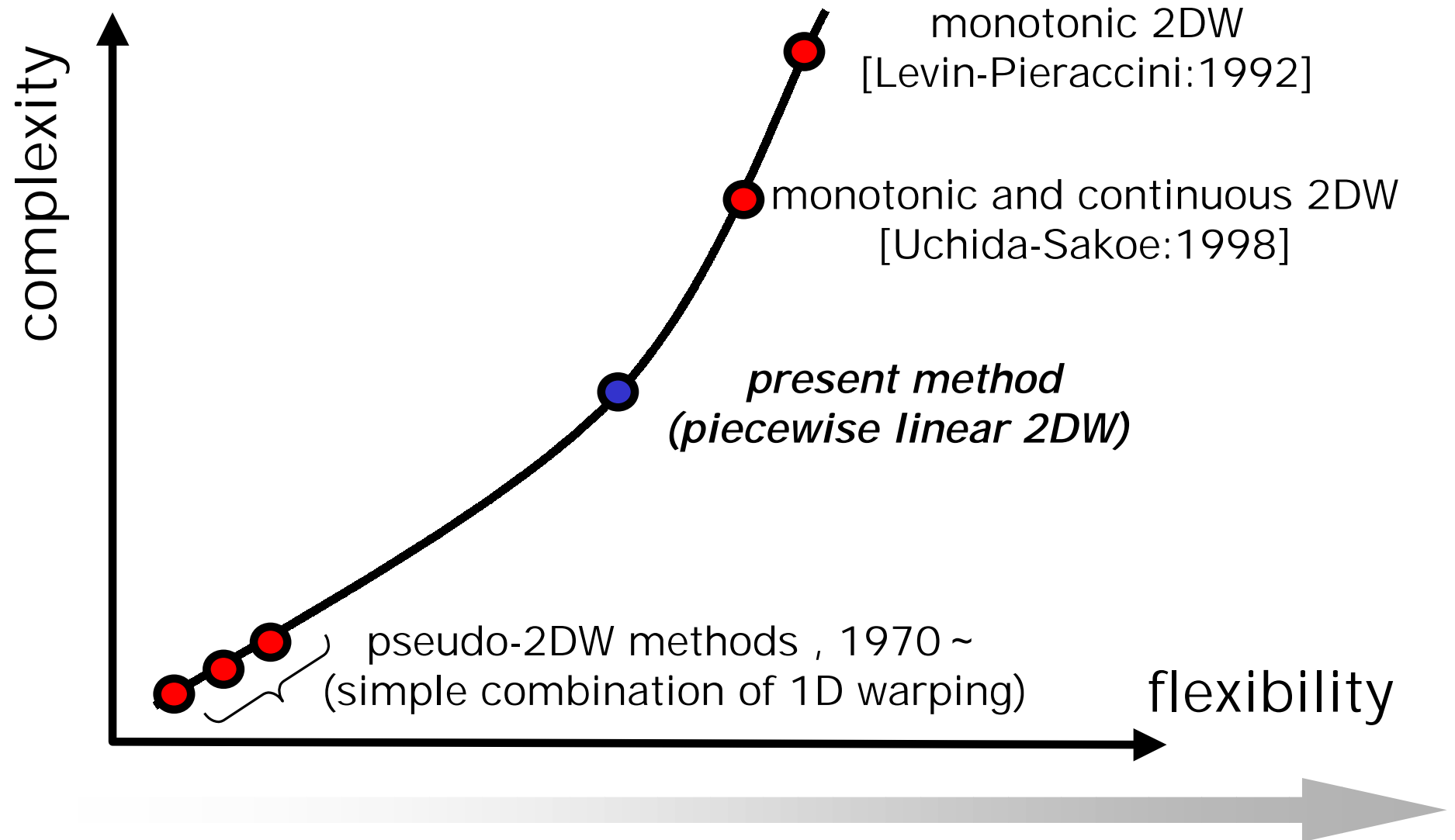
## Discussion (3) : Why DP?

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- ▶ Far less complexity than the exhaustive search
- ▶ Numerical stability
- ▶ Wide variety of criterion functions and constraints
- ▶ Global optimality
- ▶ Acknowledged performance on nonlinear time-warping for speech recognition



## Related Work : Other DP-Based 2DWs



# Future View

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- ▶ Automatic pivot placement strategy
- ▶ Non-uniform constraints
- ▶ Other interpolation methods
- ▶ ... and applications



# Conclusion

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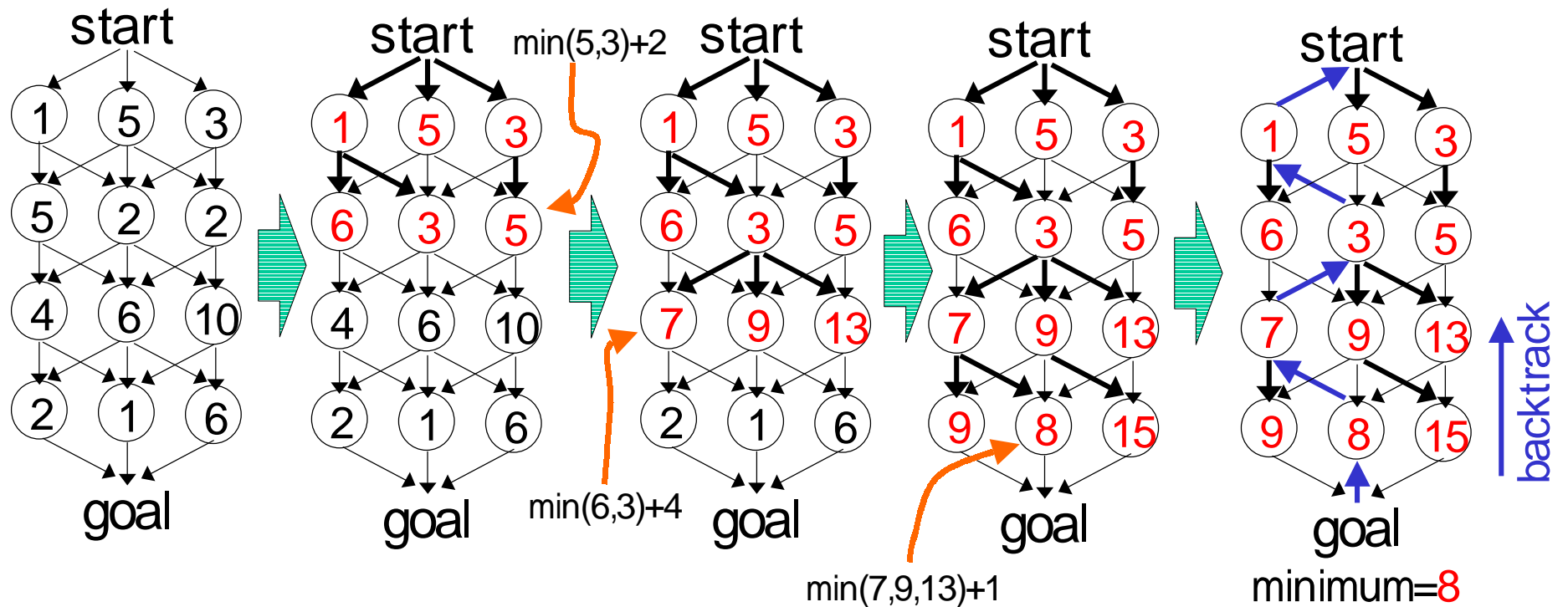
- ▶ As an elastic image matching technique, piecewise linear 2DW and its DP algorithm were proposed
- ▶ Experimental results shows that the present method provides sufficient matching





# Appendix (1) : Solve Best Path Problem by DP

PROBLEM:  
Find the minimum  
cost path



# Appendix (2): Pixelwise Organization of Algorithm

