Piecewise Linear
Two-Dimensional Warping
Elastic Image Matching

**Principle**

- 2D-2D mapping (2D warping function)

2D-2D mapping

**Two central problems**

- **Formulation** of warping function
- **Algorithm** to obtain the optimal warping function which gives the best match between given images
Formulation of Piecewise Linear 2DW (1)

- Each column of $A$ is mapped to $B$ as a broken line.
Formulation of Piecewise Linear 2DW (2)

- Monotonicity and continuity constraints for topology preservation

![Diagram showing monotonicity and continuity violations](image)

1. **Monotonicity violation**
2. **Continuity violation**
Examples (1)

![Examples](image)
Examples (2)

\[ abg \quad abg \quad abg \quad \text{warp} \quad abg \]

\[ abg \quad abg \quad abg \quad \text{warped } B \]
Examples (3)

A

B

warp

warped B
The DP Algorithm: A Graphical Representation

2DW optimization problem
- Row-wise organization
- Best path problem
- Efficiently solvable by DP
Computational Complexity

Search width \(= O(N^{2K-2})\)

\[
\begin{align*}
\text{Time complexity} & \approx O\left(N^{2K-1}(9^K + N)\right) \\
\text{Space complexity} & \approx O\left(N^{2K-1}\right)
\end{align*}
\]
Complexity Reduction to Use More Pivots

- Re-organizing the DP Algorithm
  - Use Pixel-wise organization instead of the row-wise organization
  - Still remains in exponential order of \#pivots, $K$

- Applying Beam Search
  - Pruning off less hopeful search paths
  - Suboptimal solution is obtained in polynomial time
Discussion (1): Admissible Deformation

- X-Y separable deformations:
  - Translation
  - Scaling
- 1D nonlinear deformations
- Column/row-wise deformations:
  - Bend (vertical)
  - Bend (horizontal)
  - Skew
- Truly 2D deformation: Rotation

.. and their combinations
Discussion (2): Where to Place the Pivots?

- Pivots should be placed on the points considering deformation characteristic of target.

- Distortion due to linear interpolation should also be considered.
Discussion (3): Why DP?

- Far less complexity than the exhaustive search
- Numerical stability
- Wide variety of criterion functions and constraints
- Global optimality
- Acknowledged performance on nonlinear time-warping for speech recognition
Related Work: Other DP-Based 2DWs

- Monotonic 2DW [Levin-Pieraccini:1992]
- Monotonic and continuous 2DW [Uchida-Sakoe:1998]
- Present method (piecewise linear 2DW)
- Pseudo-2DW methods, 1970~ (simple combination of 1D warping)

Graph showing the trade-off between complexity and flexibility.
Future View

- Automatic pivot placement strategy
- Non-uniform constraints
- Other interpolation methods
- ... and applications
Conclusion

- As an elastic image matching technique, piecewise linear 2DW and its DP algorithm were proposed.

- Experimental results show that the present method provides sufficient matching.
Appendix (1): Solve Best Path Problem by DP

PROBLEM:
Find the minimum cost path
Appendix (2): Pixelwise Organization of Algorithm