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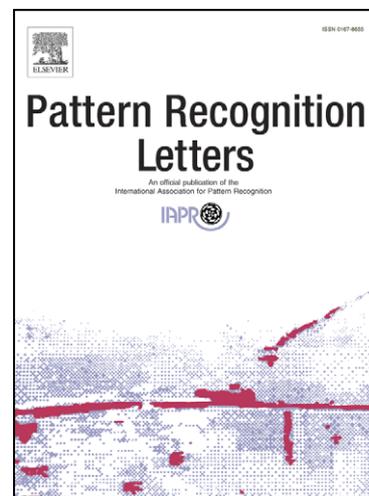
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Global Feature for Online Character Recognition

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Abstract

This paper focuses on the importance of global features for online character recognition. Global features represent the relationship between two temporally distant points in a handwriting pattern. For example, it can be defined as the relative vector of two xy -coordinate features of two temporally separated points. Most existing online character recognition methods do not utilize global features, since their non-Markovian property prevents the use of the traditional recognition methodologies, such as dynamic time warping and hidden Markov models. However, we can understand the importance of, for example, the relationship between the starting and the ending points by attempting to discriminate “0” and “6”. This relationship cannot be represented by local features defined at individual points but by global features. Since $O(N^2)$ global features can be extracted from a handwriting pattern with N points, selecting those that are truly discriminative is very impor-

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tant. In this paper, AdaBoost is employed for feature selection. Experiments prove that many global features are discriminative and the combined use of local and global features can improve the recognition accuracy.

Keywords: Online character recognition, Feature extraction, Global shape description, Feature selection

1. Introduction

In recent years, interest in online character recognition has increased because of the rapid adoption of smartphones and tablet PCs. Online character recognition is a better method than software keyboards for such small devices. An online character recognition method is needed that offers better recognition performance and is more robust against the variation in input environment.

In the long history of handwritten character recognition, feature extraction has been one of the most important topics. Any handwritten character is comprised of one or more strokes and thus has a peculiar structure unlike those of visual objects. Feature extraction as the representation of character strokes is, therefore, an important stage and affects the classification accuracy.

In this paper we tackle the unsolved problem of feature extraction; that is, how important are *global features* of character strokes for character recognition?¹ Prior to introducing the idea of global features, let us start with an

¹This paper extends (Mori et al., 2012) with an improved algorithm for feature selection, additional empirical results, and an in-depth analysis of the performance of global features compared to local features.

17 explanation of *local features*. Since a character stroke is the trajectory (of a
18 pen), its local features are defined as sequences of local parts of the trajectory.
19 The most basic local feature is xy -coordinates at each point on the stroke
20 (that is, the position of the pen-tip at each time step). Another popular local
21 feature is the local direction feature (Tappert et al., 1990; Bahlmann, 2006),
22 which is derived as the relative vector of two adjacent points. Moreover,
23 dynamic features that consider handwriting movements such as the veloc-
24 ity (e.g. Plamondon et al., 1993) or other biomechanics (e.g. Plamondon and
25 Guerfali, 1998) have been proposed (Bezine et al., 2003; Kherallah et al.,
26 2008). These methods use Beta modeling and elliptical trajectory modeling
27 to analyze handwriting, and use the parameters defined in each modeling
28 method as features for online digit recognition. Therefore, they can be re-
29 garded as not only local features but also medium-sized shape descriptions.

30 In contrast, the global features examined in this paper capture the global
31 structure of character strokes. The most basic definition of a global feature
32 is a numerical representation of the relationship between two temporally
33 separated points. An example is the positional relationship between the two
34 end-points of “0”.

35 We define a global feature as the relative vector between arbitrary point
36 pairs on the stroke. In spite of its simple definition, global features have high
37 potential to represent various key characteristics of character strokes. In
38 the above example of “0”, its starting and ending points are spatially close
39 to each other. This closeness is represented directly by the global feature
40 between those points, an ability not offered by local features.

41 Most online character recognition methods have used only local features

42 such as the xy -coordinate feature and the local direction feature to represent
43 character strokes. This might be derived from the fact that online character
44 recognition methods are often based on dynamic time warping (DTW) or
45 hidden Markov models (HMM) using sequences of local features. Both of
46 them require that the problem have the Markovian property. Unfortunately,
47 the use of global features clearly violates this Markovian constraint because
48 they deal with the relationship between distant points. Consequently, to the
49 best of the authors' knowledge, global features have not fully been investi-
50 gated and thus the superiority of global features over local features has not
51 been confirmed.

52 The main contributions of this paper can be summarized as follows: first,
53 we prove that global features are often more discriminative than local fea-
54 tures. This is proved through an experiment on automatic feature selection
55 using the AdaBoost-based machine learning framework. In the experiment,
56 an online numeral dataset is utilized for training a classifier. We also observe
57 the global features that are selected and show that the global features that
58 link separated points are certainly important for character shape description.
59 Second, through a recognition experiment on a test dataset, we prove that
60 the use of global features yields better recognition accuracy.

61 We examine the performance of global features for the online character
62 recognition task throughout this paper. An important note is that if global
63 features are useful in the online classification task, they will also be useful in
64 the offline classification task. In fact, nowadays, offline recognition methods
65 can employ techniques created for online recognition. For example, in (Mori
66 et al., 1992), offline recognition is performed by extracting local online fea-

67 tures of strokes by image processing. More dramatically, it is possible to
68 use some stroke recovery method to convert an offline pattern into an online
69 pattern.

70 The rest of this paper is organized as follows: Section 2 reviews related
71 works in feature extraction. Section 3 explains the global feature approach.
72 Experiments and results are reported and discussed in Section 4. Section 5
73 derives conclusions and future works.

74 2. Related Work

75 Most online character recognition methods use local features, such as xy -
76 coordinate features and local direction features, as noted in Section 1. Those
77 local features are sequential and thus traditional Markovian methodologies
78 such as DTW and HMM can be applied. However, this fact does not prove
79 that the handwriting process is purely Markovian. While writing a character
80 pattern, we are usually watching not only the pen-tip but also the stroke
81 shape of the written part. This process is totally non-Markovian and helpful
82 in avoiding confusion between “0” and “6”.

83 In spite of the expected merit of global features, only a few trials have
84 been made on utilizing them in the online character recognition task. One
85 study examined the relative stroke position feature (Shin et al., 1999; Ota
86 et al., 2007). This feature represents inter-stroke relationship and thus does
87 not represent the stroke shape. Another trial is the star feature (Mandalapu
88 and Krishna, 2007) which is based on an eight-directional representation (i.e.,
89 a quantized representation) of the entire character stroke; it can be seen as
90 a online version of the classical Sonde method (Johnson, 1956) for offline

91 character recognition.

92 The trial by Izadi and Suen (Izadi and Suen, 2009) is the work closest
 93 to our study. They proposed a feature, called relational context, that com-
 94 putes the relative pairwise distances and angles between arbitrary point pairs.
 95 Their trial, however, was merely a preliminary evaluation of the usefulness of
 96 global features. They used online patterns, each of which was re-sampled to
 97 just 6 points, and all ${}_6C_2$ pairs were used for extracting ${}_6C_2$ global features.
 98 As shown by the feature selection experiment in Section 4, our method has no
 99 need to use all such pairs in extracting useful global features. In other words,
 100 we reveal that each character class has its own important global structure;
 101 this important fact is not examined in (Izadi and Suen, 2009).

102 In offline handwritten character recognition (OCR), we can find features
 103 representing the relationship *spatially* distant points, i.e., pixels. One clas-
 104 sical example is the so-called crossing features (Johnson, 1956; Glucksman,
 105 1967), which describe the relationship between a target pixel and spatially
 106 distant strokes. Features that extract relative angle and relative position
 107 from spatially adjacent strokes have also been proposed (Mori et al., 1998).

108 3. Global features

109 We denote a handwriting pattern as the sequence, $p_1, \dots, p_n, \dots, p_N$,
 110 where p_n is the xy -coordinate feature of the n -th trajectory point. We assume
 111 that the number of trajectory points, N , is fixed for all patterns by using
 112 some resampling procedure. The coordinate feature, p_n , is considered as a
 113 local feature. Another local feature candidate is the local direction feature,
 114 which is defined as $p_n - p_{n-1}$. The experiments in Section 4 uses them as

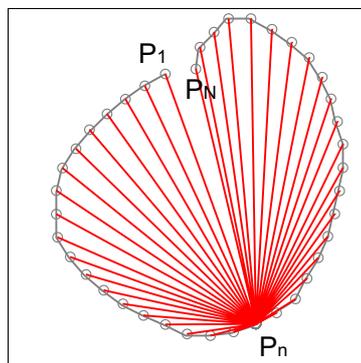


Figure 1: Global feature extraction.

115 typical local features.

116 The global feature used in this paper is simply defined as the relative
 117 vector between two temporally separated points n, n' , that is, $p_n - p_{n'}$. Fig-
 118 ure 1 shows the global features $p_n - p_{n'}$ from n to $n' \in \{1, \dots, N\}$. It should
 119 be noted that the global features include the local direction features as a
 120 special case. Hereafter, we assume that $n > n'$ for choosing one of the two
 121 reciprocal features, $p_n - p_{n'}$ and $p_{n'} - p_n$. Thus, for a handwriting pattern
 122 with N trajectory points, there are ${}_N C_2$ possible global features.

123 Figure 2 (a) shows an example where the global feature $p_N - p_n$ is discrim-
 124 inative for two classes, “2” and “3”. It is also shown that the other global
 125 feature, $p_n - p_1$, is almost the same in both classes and thus less discrimi-
 126 native; however, global features between more distant points, say $p_N - p_1$,
 127 become more discriminative and thus we can expect a set of global features
 128 to yield correct discrimination between “2” and “3”. Figure 2 (b) shows the
 129 global feature $p_N - p_1$ of “0”. We can easily expect that this global feature
 130 can well discriminate “0” from “6”.

131 As indicated by Fig. 2 (a), different global features have different dis-

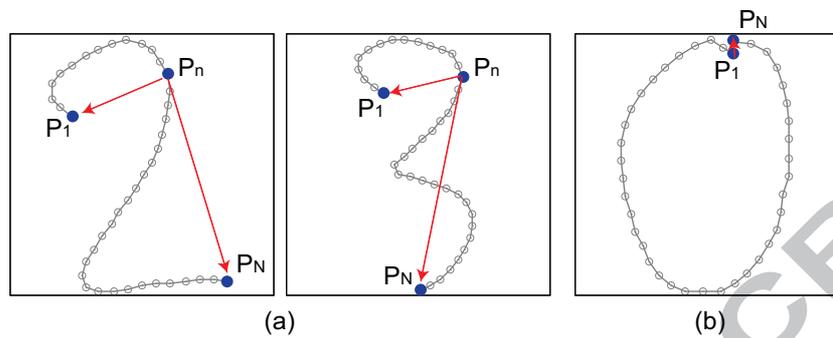


Figure 2: Potential of global features. (a): The local features around p_n do not have any discriminative power unlike the global features related to p_n . (b): The global features can regulate the relationship between two temporally separated points such as the starting and ending points.

132 criminative power. In other words, we do not need to use all of the ${}_N C_2$
 133 global features. This observation suggests the necessity of some appropriate
 134 feature selection procedure. Section 4 adopts AdaBoost for feature selec-
 135 tion (Freund and Schapire, 1997); AdaBoost has been utilized as not only a
 136 machine-learning method for training a classifier but also as a feature selec-
 137 tion method. Thus, by using AdaBoost, we can select the global features that
 138 offer better classification and obtain a strong classifier for online character
 139 recognition.

140 4. Experimental evaluation of the importance of global features

141 In this section, the importance of global features is examined through two
 142 experiments; the first experiment addresses feature selection and the other
 143 recognition. They are detailed below.

144 4.1. *Experimental setup*

145 Online numeral samples from the UNIPEN database (Guyon et al., 1994)
 146 were used in our experiment. The database contains 31,386 samples of the
 147 10 numeral classes (“0”-“9”). 90% of the samples (28,248) were used for
 148 training and the remaining 10% samples (3,138) were used for testing. In
 149 preprocessing, each sample was linearly normalized to 128×128 while keeping
 150 its original aspect ratio and then re-sampled at $N = 40$. Finally, N local
 151 features of xy -coordinates and ${}_N C_2$ global features including local direction
 152 features were extracted from each sample.

153 We set subclasses for each class to deal with changes in writing order
 154 and direction. We exploited the k -means algorithm to cluster the training
 155 samples. Finally, we used 22 subclasses for the 10 numeral classes; 2 ~ 3
 156 subclasses were prepared for each class. Therefore, we set $P = 22$ one-vs.-
 157 others classifiers in this paper.

158 4.2. *Feature selection experiment*

159 The first experiment uses AdaBoost (Freund and Schapire, 1997) to de-
 160 termine which features are most useful among all local features and global
 161 features in terms of discrimination. AdaBoost is an iterative learning scheme
 162 and iteratively selects a classifier, called a weak learner. After M iterations,
 163 we have M weak learners. The final result, the so-called strong classifier, is
 164 realized by weighting and summing the outputs of the M weak classifiers.
 165 In the experiment, the m -th weak classifier was designed to use only the
 166 single feature selected in the m -th iteration to provide a classification result.
 167 Thus, this iterative learning scheme can be considered as a feature selection
 168 process. A feature, as a weak learner, that is selected at an earlier step

169 can be regarded as a more important feature for classification. Thus, under
170 the condition that both global and local features are provided to AdaBoost,
171 the selection of more global features than local features in the first few it-
172 erations means that the global features are experimentally confirmed to be
173 more important than local features. The order of selection also reveals the
174 importance of features. We, therefore, can observe which global features are
175 important for each class from the selection result. The basic algorithm of
176 AdaBoost is described in the Appendix.

177 Since AdaBoost is a two-class classifier, we train P “one-vs.-others” clas-
178 sifiers. In the following experiments of numeral recognition, P was fixed not
179 at 10 but at 22. This is because several numeral classes had a large diversity
180 due to changes in writing order and direction and so were divided into 2 or 3
181 subclasses. As the weak learner, we used a linear classifier based on the prin-
182 cipal of nearest-neighbor to the weighted centroid for each of two classes (i.e.,
183 the target class and the “others” class). In our AdaBoost implementation,
184 the best weak learner was selected from ${}_N C_2$ weak learner candidates at each
185 iteration. Moreover, for creating better candidates, a random perturbation
186 process was introduced for each feature as a weak learner. Specifically, the
187 position of the weighted centroid was perturbed by adding a small random
188 vector. By using 10 different random vectors, each weak learner candidate
189 were evaluated 10 times and then one with the minimum error rate among
190 ${}_N C_2$ weak learners was selected as the weak learner at that iteration. Here we
191 mention the reason to introduce the random perturbation and its effective-
192 ness. In our initial experiment, when the random perturbation was not used,
193 the iteration for selecting features as weak learners terminated at earlier steps

194 in the learning stage. The reason is the simplicity of the weak classifier used.
195 For examples, the learning on the xy -coordinate feature as the lower dimen-
196 sional feature terminates at less than 30 steps for some classes. This gives
197 the lower recognition accuracy for the xy -coordinate feature, and we could
198 not fairly evaluate the effectiveness of our proposal. Therefore, we needed
199 to obtain more features for stronger classifiers and exploited the random
200 perturbation for more fair comparison among features and higher recogni-
201 tion results. The superiority among features based on the learning with no
202 random perturbation was similar to that with the random perturbation.

203 Figure 3 shows the features selected in the first 20 iterations of the “0”-
204 vs.-others classifiers. In Figure 3 (a), selected xy -coordinate features do not
205 appear to express the characteristic shape of “0”. In Figure 3 (b), local
206 direction features accounted for about half the total number of features se-
207 lected. This means that local direction features are not clearly superior to
208 xy -coordinate features. In contrast, from Figure 3 (c), the global features
209 representing the relationship between distantly-positioned point pairs were
210 selected as the most important features in the top 3. The feature defined
211 between the point near the start and the point near the end was also selected
212 as one of the top 5 features. Among the 10 numerals, “0” has a circular
213 shape and the start and end points are close to each other. This means that
214 these relationships are important for characterizing the shape of “0” and
215 our feature describes this global information effectively. Figure 3 (c) also
216 shows that global features were more frequently selected for “0” within the
217 first 20 iterations. This tendency confirms that most selected global features
218 represent the relationship between pairs of separated points; that is, they

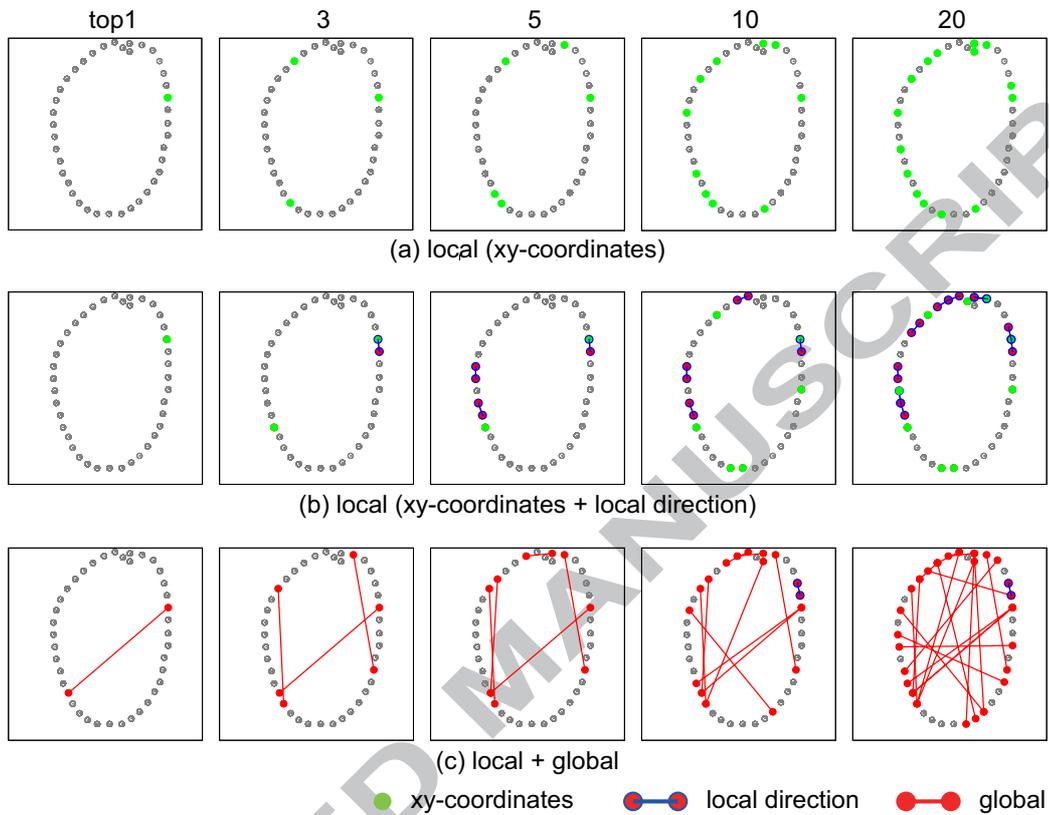


Figure 3: Selected features for “0”.

219 represent that “0” is a circular pattern and has a large empty internal space.

220 Figure 4 visualizes the distributions of the global features selected in the
 221 first and second iteration under the AdaBoost framework. It seems that the
 222 two classes, “0” and the “others” class, are well discriminated by a simple
 223 linear classifier which validates the use of AdaBoost for feature selection.

224 Figure 5 shows the features selected in the first 10 iterations in each class.
 225 Global features were frequently selected not only for “0” but also the other
 226 classes. This fact is shown more clearly in Figure 6 which depicts the order

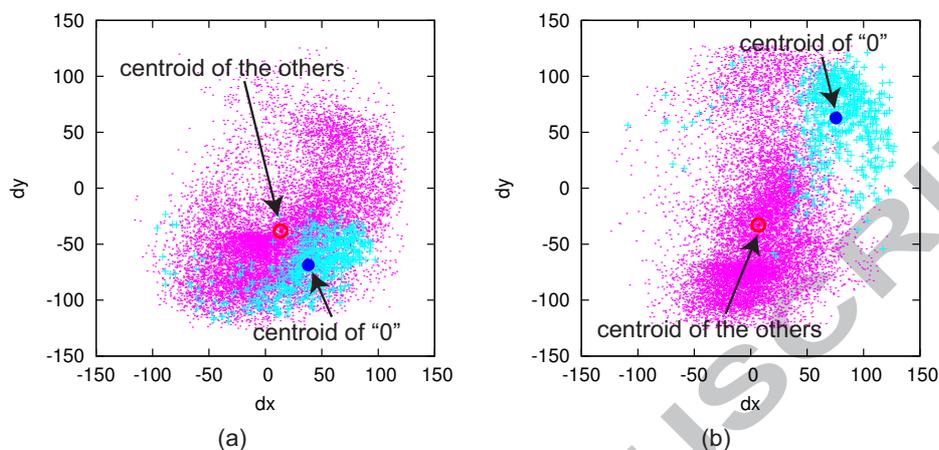


Figure 4: Distribution of the global feature vector. (a) Feature selected at the first iteration (leftmost feature of Fig. 3 (c)) for “0”-vs.-others classifier. (b) Feature selected at the second iteration.

227 of features selected in the first 20 iterations. Here the global feature excludes
 228 local direction. In most classes, global features are selected more frequently
 229 in the first 20 iterations than local ones. These results prove the stability
 230 and the discrimination power of global features.

231 4.3. Recognition experiment

232 In this experiment the P one-vs.-others classifiers are trained by Ad-
 233 aBoost using only global features, only local features, or both features to-
 234 gether. Next, each test sample is subjected to the P classifiers and then
 235 classified into the class indicated by the subclass with the highest score. If
 236 the classifiers trained using the global features outperform those trained using
 237 the local features, an experimental proof of the importance of global features
 238 can be posited.

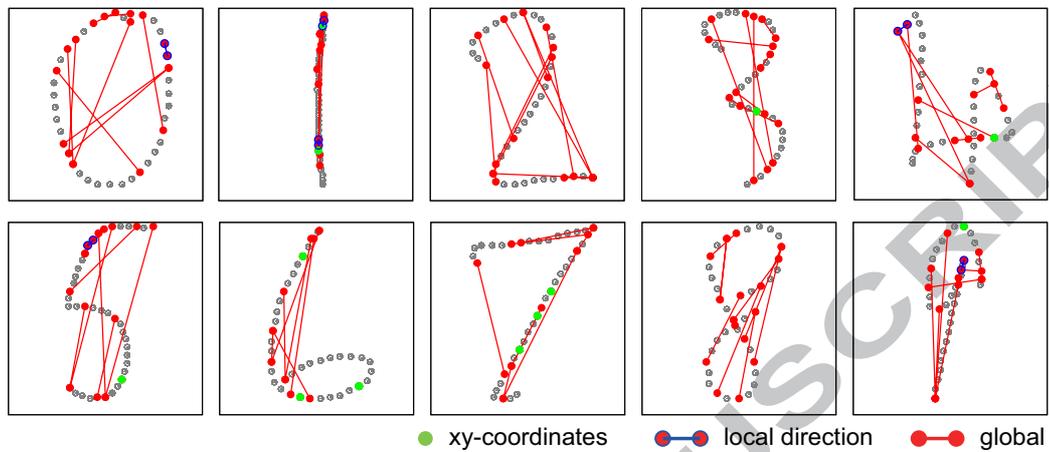


Figure 5: The first 10 features selected from among global and local features.

239 Figure 7 shows the classification accuracies during feature selection, that
 240 is, AdaBoost training. Global features outperformed the local features, and
 241 their combination yields better accuracy. These results suggest that the
 242 global features extract more important information of each class than the
 243 information captured by the local features. Here, note that the training
 244 iteration stopped when AdaBoost could not find any other weak learner that
 245 could improve overall accuracy. Also note that each feature could be selected
 246 repetitively. Therefore, the number of selected feature, M , can exceed the
 247 number of original features.

248 Figure 8 shows the recognition accuracy of the test samples when sub-
 249 jected to 10-class numeral classification. As expected from the accuracy of
 250 the training step (Figure 7), the combination of local and global features
 251 achieves the best accuracy (97.3%) with the use of just global features a
 252 close second. These results prove that our global features are more effective
 253 for online character recognition and have more discrimination power than

254 local features. We note that our current result (97.3% accuracy) is 1.6%
255 lower than the best past trial (98.9%) (Ratzlaff, 2003). The main purpose of
256 this paper is to clearly prove the relative superiority of global features over
257 local features. Thus, our experimental setup has a lot room for improvement.
258 For example, we can introduce more subclasses as used in (Ratzlaff, 2003).
259 Many post-processing techniques, such as 1-vs.-1 classification between an
260 ambiguous class pair, also can be introduced.

261 Figure 9 shows examples that were recognized erroneously by the local
262 features but recognized correctly by the global features. These examples
263 show that global features can effectively extract crucial information for dis-
264 criminating the focused class from the other classes, and provide robustness
265 against the shape distortion common in handwriting. In contrast, local fea-
266 tures lost their power of discrimination in the presence of shape distortion.
267 For example, the local direction features in the upper part of the misrecog-
268 nized “3” have similar directions to local direction features in the upper part
269 of “5”. On the other hand, our method describes the global structure of
270 the upper part of “3” and identifies the global features that can effectively
271 discriminate “3” from “5”.

272 Figures 10 are examples of erroneous recognition by global features. The
273 causes of errors are (a) different stroke order or different stroke direction that
274 are not contained in the training samples used for each subclass, (b) similar
275 shape to another class, (c) strong shape distortion, and (d) mislabeling. To
276 deal with cause (a), more subclasses for each category are needed. A classifier
277 that can discriminate ambiguous class pairs could offset cause (b). Cause (c)
278 could be tackled by non-linear normalization techniques as pre-processing.

279 **5. Conclusion**

280 We have described the importance of the global features of character
281 strokes and confirmed it in two experiments. A global feature is defined as a
282 relative vector between arbitrary separated pairs of two points on a charac-
283 ter stroke. Even though our proposal is very simple, it can represent various
284 characteristics that can never be represented by local features. For example,
285 a global feature can represent the very small separation between the start
286 and end points of “0”. Conventional online recognition frameworks, such
287 as DTW and HMM, ignore or neglect “non-Markovian” features, i.e. global
288 features, because their algorithmic principle is overly constrained. Thus, one
289 contribution of this paper is to recommend that researchers re-consider the
290 usefulness of “non-Markovian” features. Our experiments used an online
291 character recognition task with numeral samples from the UNIPEN dataset.
292 The feature selection experiment with the AdaBoost-based machine learn-
293 ing framework revealed that global features were more important in terms
294 of discrimination power than local features; global features are crucial for
295 identifying the characteristics of each class. The recognition experiment also
296 proved that global features yield better classification accuracy not only for
297 training samples but also for test ones.

298 Future works include the theoretical extension of our global feature. the
299 global feature proposed in this paper is just a relative vector, i.e., a difference
300 vector, between two xy -coordinates of a point pair; this can be extended to a
301 difference vector of another local feature. Our feature can also be extended
302 to deal with the relationship between three or multiple points. For the on-
303 line recognition setup, introduction of the time warping function (other than

304 DTW) is also promising. For higher recognition accuracy, it is straightfor-
305 ward to prepare several classifiers for each class in order to deal with stroke
306 order. To observe the important global features of different character sets
307 such as Latin alphabets and Chinese characters is also interesting.

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354 Appendix

355 The algorithm of AdaBoost (Freund and Schapire, 1997) is described as
 356 follows:

357

358 Given:

359 • Training set: $(x_1, y_1), \dots, (x_s, y_s); x_i \in \chi, y_i \in \{-1, 1\}$

360 • Number of iterations: M

361 Initialize weights: $D_1(i) = \frac{1}{s}, i = 1, \dots, s$

362 For $t = 1, \dots, M$:

363 1. Find the weak learner $h_t : \chi \rightarrow \{1, -1\}$ with minimum error rate ϵ_t with
 364 respect to the distribution D_t . Here $\epsilon_t = \sum_{i=1}^s D_t(i)I(y_i \neq h_t(x_i))$ and
 365 I is the indicator function.

366 2. if $|0.5 - \epsilon_t| \leq \beta$ then stop. Here β is a threshold.

- 367 3. Choose $\alpha_t \in \mathfrak{R}$, here $\alpha_t = \frac{1}{2} \ln \frac{1-\epsilon_t}{\epsilon_t}$
- 368 4. Update $D_{t+1}(i) = \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t}$. Here Z_t is the normalization fac-
- 369 tor $\sum_i D_t(i) \exp(-\alpha_t y_i h_t(x_i))$ and D_{t+1} is a distribution.
- 370 Output the strong classifier: $H(x) = \text{sign}(\sum_{t=1}^M \alpha_t h_t(x))$

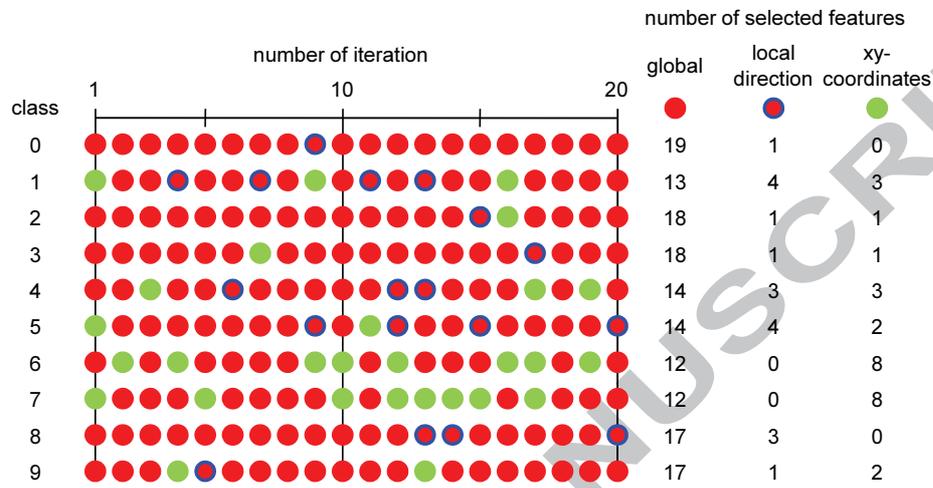


Figure 6: Features selected in the first 20 iterations.

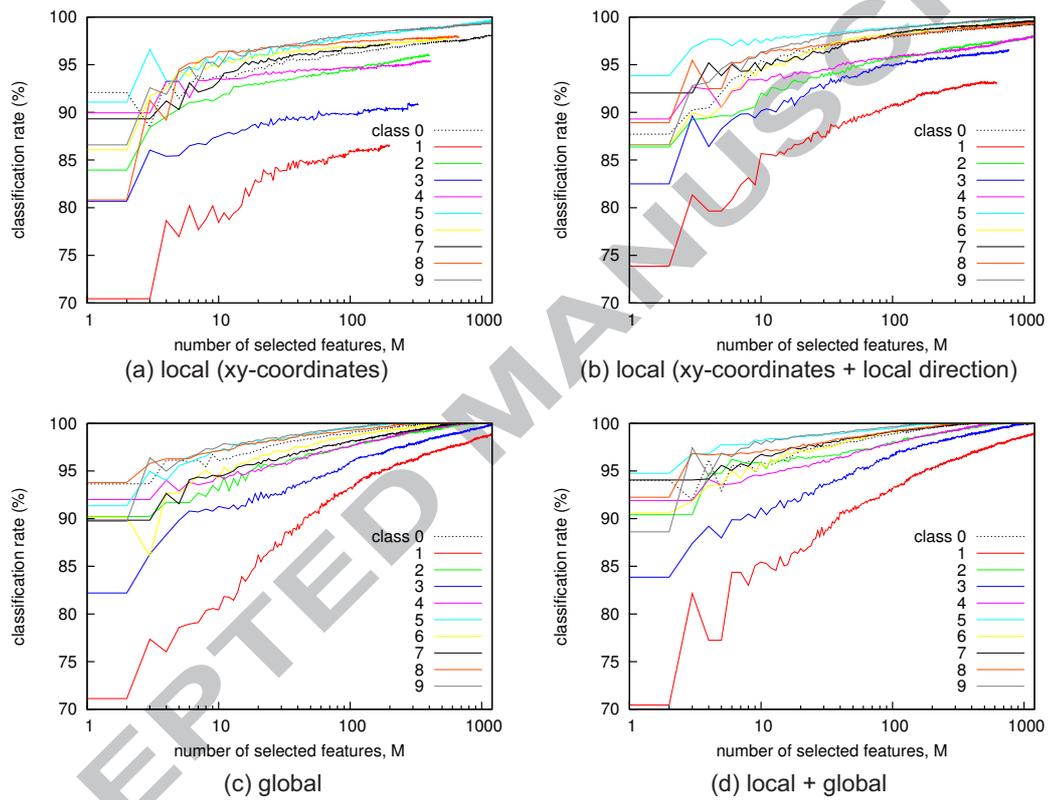


Figure 7: Classification accuracy for training data by each one-vs.-others classifier.

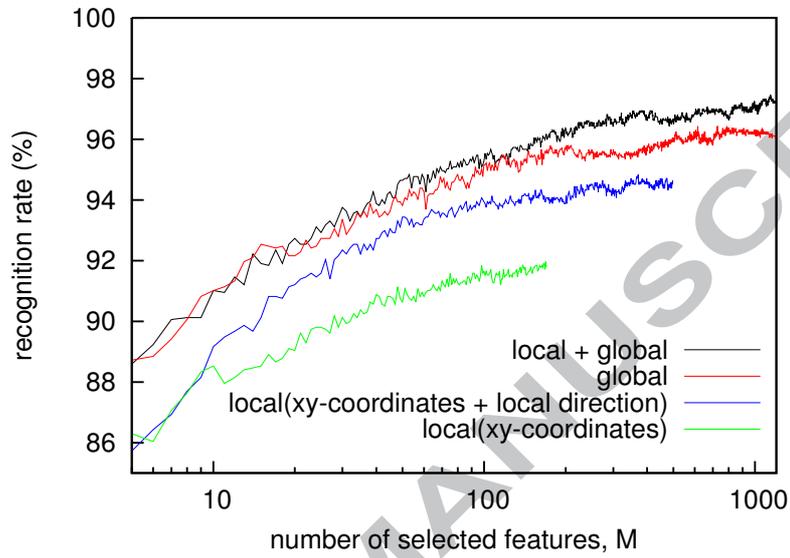


Figure 8: Recognition accuracy for test data.

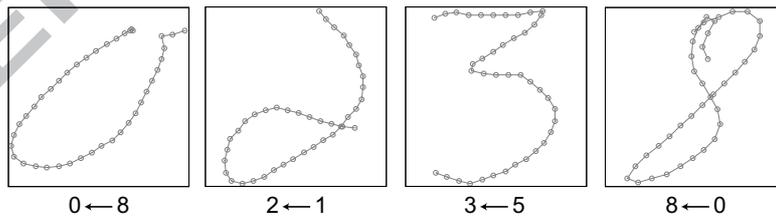


Figure 9: Patterns recognized erroneously and correctly by local and global features, respectively.

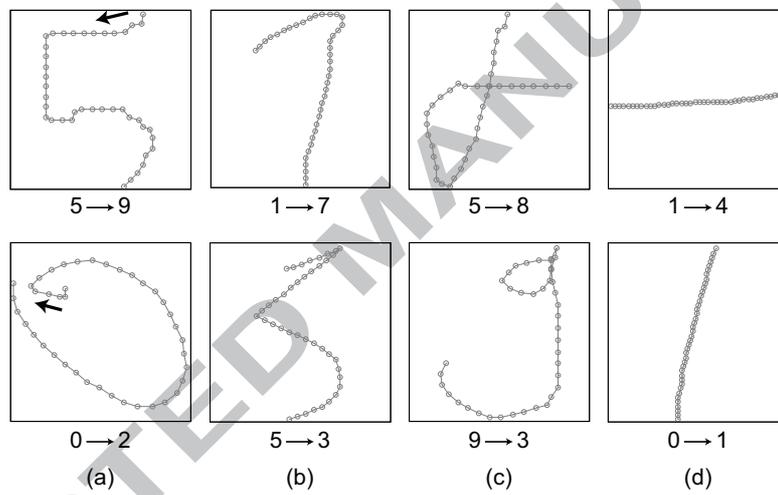


Figure 10: Patterns misrecognized by all four features.

We propose global features that extract global structure of online character pattern.

Global features represent the relationship between distantly-positioned points.

We validate global features through feature selection and recognition experiments.

Global features are more often selected as important classifiers than local features.

Global features achieve higher recognition rates than local features.

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